Nonlinear optics and quantum communication, polarization optics and quantum computation

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Agenda

Linear Optics

Maxwell's equation, Maxwell's wave equation (a free space)

$$\begin{aligned} \nabla \cdot \boldsymbol{E} &= 0, \\ \nabla \cdot \boldsymbol{B} &= 0, \\ \nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t}, \\ \nabla \times \boldsymbol{B} &= \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \end{aligned} \quad (1)$$

Gauss' law,
Gauss' law in magnetic system (non existence of magnetic monopole),
Faraday's law,
Ampere's law

$$\nabla^2 \boldsymbol{E} = \frac{1}{c^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2} \quad (2)$$

Maxwell's wave equations

$$\epsilon_0 = 0.854 \times 10^{-12} F/m,$$
$$\mu_0 = 4\pi \times 10^{-7} H/m$$

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)} = \boldsymbol{E}_0 e^{i\phi(\boldsymbol{r},t)} \quad (3)$$

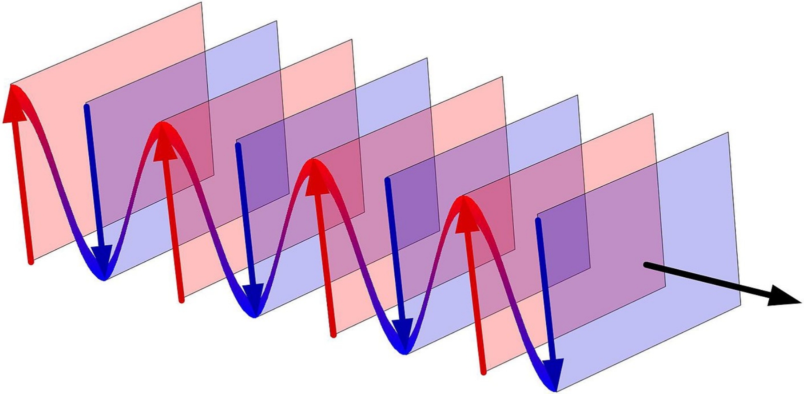
Plane wave solution

$$\omega/k = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 299,792,458 m/s,$$
$$Z_0 = \frac{|\boldsymbol{E}_0|}{|\boldsymbol{H}_0|} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.730313668(57)\Omega$$

"-" sign suggests that the wave is moving in the positive direction

Plane wave

Surface of constant phase at any time t (phase front, wave front) is a plane, since $\boldsymbol{k} \cdot \boldsymbol{r} = const$ is equation of a plane, where $\boldsymbol{k}$ is a normal vector to the plane, and $\boldsymbol{r}$ is the position vector of a point in the plane.



Monochromatic vs non-monochromatic wave

$$\phi(\boldsymbol{r},t) = \boldsymbol{k}\cdot\boldsymbol{r} - \omega_0 t, \quad \text{Frequency} = -\frac{\partial\phi}{\partial t} = \omega_0 \tag{4}$$

$$\phi(\boldsymbol{r},t) = \boldsymbol{k}\cdot\boldsymbol{r} - \omega_0 t \pm \delta(t), \quad \text{Frequency} = -\frac{\partial\phi}{\partial t} = \omega_0 \pm \frac{\partial\delta(t)}{\partial t} \tag{5}$$

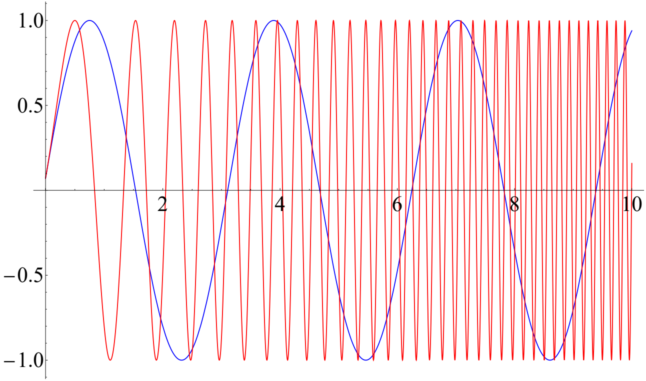


Figure – Monochromatic wave vs chirped wave ($\frac{\partial\delta(t)}{\partial t} \propto t$).

*Non-monochromatic waves

Using time-only scalar representation $E = E_0 e^{-i\omega_0 t}$. Fourier transform of sine wave is δ-function (idealized monochromatic).

A non-monochromatic wave contains multiple frequency components within its spectrum (Fourier transform of a wave). Waveform that changes over time (or space) results in (spatial) spectral broadening or (spatial) frequency modulation, which leads to multiple (spatial) frequency components in its spectrum. Time (spatial)-varying amplitude, frequency, or phase are the cases.

Examples of non-monochromatic waves include (spatial) chirped pulses, (spatial) frequency-modulated signals, and (spatial) wave packets.
*Hereinafter, we discuss temporal frequency!!!

Quasi-monochromatic pulse : $E(t) = E_0(t)e^{-i\omega_0 t}$, ω_0 is a carrier frequency, $E_0(t)$ — envelope or modulation, $e^{-i\omega_0 t}$ — a carrier wave. These pulses' frequency domain representation : Fourier transform $\mathcal{F}[E(t)] = \mathcal{E}(\omega) = \int_{-\infty}^{\infty} E(t)e^{i\omega t}dt$. Inverse Fourier transform $\mathcal{F}^{-1}[\mathcal{E}(\omega)] = E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathcal{E}(\omega)e^{-i\omega t}dt$.

*Quasi-monochromatic pulse

Gaussian pulse $E_0(t) = E_0 e^{-\frac{t^2}{\tau^2}}$, where τ is a temporal width of a pulse. Knowing that Gaussian integral (normal distribution integral) $\int\limits_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$, we get $\mathcal{E}(\omega) = \frac{E_0\sqrt{\pi}}{\tau} e^{-\frac{(\omega-\omega_0)^2\tau^2}{4}}$.

1. The spectrum associated with Gaussian pulse is also Gaussian, centered on the carrier frequency ω_0,
2. Considering τ in the numerator of $\mathcal{E}(\omega)$ and in the denominator of $E(t)$, there is a reciprocal relationship : a signal that wider temporally, is narrower spectrally and vice versa (**works for non-Gaussian pulses too ! ! !**). This reflects a fundamental property of signals known as the **time-frequency duality principle**, which is a consequence of the **uncertainty principle** $\Delta t \cdot \Delta\omega \geq \frac{1}{2}$ in signal processing ! ! ! Very short pulse of light would have to contain many-many colors. For a monochromatic signal, we need very wide pulse in time, something approaching pure sine wave.

*Quasi-monochromatic pulse

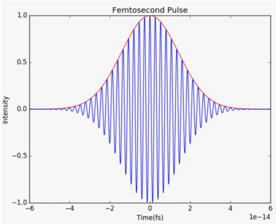
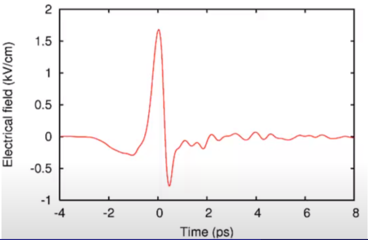
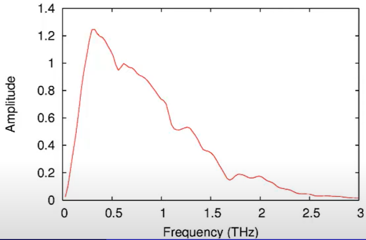


Figure – Blue : Gaussian (envelope) femtosecond pulse. Red : picosecond pulse.





Maxwell's equation (general case)

$$
\begin{aligned}
\nabla \cdot \boldsymbol{E} &= \frac{\rho_f + \rho_b}{\epsilon_0} = \frac{\rho_f}{\epsilon_0} - \frac{1}{\epsilon_0}\nabla \cdot \boldsymbol{P} \\
&\quad (\Leftrightarrow \nabla \cdot \boldsymbol{D} = \rho_f), \\
\nabla \cdot \boldsymbol{B} &= 0, \\
\nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t}, \\
\nabla \times \boldsymbol{H} &= \boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}; \\
\boldsymbol{D} &= \epsilon_0 \boldsymbol{E} + \boldsymbol{P}, \quad \boldsymbol{B} = \mu_0 \boldsymbol{H} + \boldsymbol{M}, \\
\boldsymbol{j} &= \sigma_0 \boldsymbol{E} + \sigma_1 \boldsymbol{E}^2 + \dots, \\
\boldsymbol{P} &= \epsilon_0 \chi^{(1)} \boldsymbol{E} + \epsilon_0 \chi^{(2)} \boldsymbol{E}^2 + \dots, \\
\boldsymbol{M} &= \mu_0 \chi_m^{(1)} \boldsymbol{H} + \mu_0 \chi_m^{(2)} \boldsymbol{H}^2 + \dots .
\end{aligned} \tag{6}
$$

Gauss' law,
Gauss' law in magnetic system (non existence of magnetic monopole),
Faraday's law,
Ampere's law ;
material equations (constitutive relations)
(ρ : the electric charge density,
$\boldsymbol{j}$: the current density
$\boldsymbol{D}$: the displacement field (electrical flux density)
$\boldsymbol{H}$: the magnetizing field (magnetic field strength)
$\boldsymbol{B}$: the magnetic field (magnetic flux density)
$\boldsymbol{E}$: the electric field (strength))

Maxwell's equation (general case)

If nonlinear and anisotropic media in $\boldsymbol{P}(\boldsymbol{E})$ — $\chi^{(1)}$ and $\chi^{(i)}$ $(i \geq 2)$ are linear and ith order nonlinear dielectric susceptibility tensors, correspondingly.
If nonlinear and anisotropic media in $\boldsymbol{M}(\boldsymbol{H})$ — $\chi_m^{(1)}$ and $\chi_m^{(i)}$ $(i \geq 2)$ are linear and ith order nonlinear magnetic susceptibility tensors, correspondingly.
If nonlinear and anisotropic media in $\boldsymbol{j}(\boldsymbol{E})$ — σ_0 and σ_i are linear and ith order nonlinear conductivity tensors, respectively.

Linear and isotropic and non-magnetic media

$$\boldsymbol{P} = \epsilon_0 \chi^{(1)} \boldsymbol{E}, \quad \boldsymbol{D} = \epsilon_0 (1 + \chi^{(1)}) \boldsymbol{E} = \epsilon_0 \epsilon_r \boldsymbol{E} = \epsilon \boldsymbol{E}, \quad \epsilon_r = n^2, \tag{7}$$

$\chi^{(1)}$ is dielectric susceptibility, ϵ_r is relative permittivity, n is refractive index.

*Types of electric and magnetic materials (linear medium)

$P = \chi E$

χ is the dielectric susceptibility, a measure of how much a dielectric material can become polarized under the influence of an electric field.

- $\chi > 0$ — dielectric (capacitors (конденсатор), insulators),
- $\chi \approx +0$ — non-dielectric (conductors (проводник) and insulators (изолятор, непроводник)),
- Semiconductors depending on various factors such as doping, temperature, and applied electric field, they can indeed function as capacitors, insulators, conductors, dielectrics, and non-dielectrics.

$M = \chi_m H$

χ_m is the magnetic susceptibility, a measure of how much magnetization a material will acquire in response to an applied magnetic field.

- $\chi_m > 0$ — paramagnetic,
- $\chi_m < 0$ — diamagnetic,
- $\chi_m \gg 0$ — ferromagnetic,
- $\chi_m \approx +0$ — antiferromagnetic.
- $n < 0$ — negative index metamaterials.

*Negative index metamaterials

For double-negative metamaterials (DNG) : a class of metamaterials that simultaneously exhibit negative values of both electric permittivity (ϵ) and magnetic permeability (μ) within a certain frequency range. In other words, these materials have negative values for both the real parts of their permittivity and permeability.

In a DNG medium where $\epsilon < 0$ and $\mu < 0$

$$\begin{aligned} \sqrt{\epsilon} &= \sqrt{\epsilon_r \epsilon_0 - j\epsilon''} \approx -j\left(|\epsilon_r \epsilon_0|^{1/2} + j\frac{\epsilon''}{2|\epsilon_r \epsilon_0|^{1/2}}\right), \\ \sqrt{\mu} &= \sqrt{\mu_r \mu_0 - j\mu''} \approx -j\left(|\mu_r \mu_0|^{1/2} + j\frac{\mu''}{2|\mu_r \mu_0|^{1/2}}\right), \end{aligned} \tag{8}$$

Double negative metamaterials are of particular interest because they can exhibit negative refractive index, meaning they refract light in the opposite direction compared to conventional materials. This property enables a range of novel applications in optics and electromagnetics, such as superlenses, cloaking devices, and subwavelength imaging systems.

*Negative index metamaterials

Wave impedance, also known as characteristic impedance, is a property of electromagnetic waves propagating through a medium. It represents the ratio of the electric field strength to the magnetic field strength of the wave, and it is denoted by the symbol Z or η.
Wavenumber k represents the spatial frequency of a wave, or how rapidly the phase of the wave changes with respect to position along the direction of propagation.

$$\begin{aligned} k &= \frac{\omega}{c}\sqrt{\epsilon}\sqrt{\mu} \approx -\frac{\omega}{c}|\epsilon_r|^{1/2}|\mu_r|^{1/2}\left(1 + j\frac{1}{2}\left(\frac{\epsilon''}{|\epsilon_r|\epsilon_0} + \frac{\mu''}{|\mu_r|\mu_0}\right)\right), \\ \eta &= \frac{\sqrt{\mu}}{\sqrt{\epsilon}} \approx \mu_0\frac{|\mu_r|^{1/2}}{|\epsilon_r|^{1/2}}\left(1 + j\frac{1}{2}\left(\frac{\mu''}{|\mu_r|\mu_0} - \frac{\epsilon''}{|\epsilon_r|\epsilon_0}\right)\right), \end{aligned} \tag{9}$$

where the speed of light $c = 1/\sqrt{\epsilon_0\mu_0}$ and the free-space wave impedance $\eta_0 = \sqrt{\mu_0/\epsilon_0}$.

*Negative index metamaterials

One sees that the index of refraction

$$n = \frac{kc}{\omega} = \sqrt{\frac{\epsilon}{\epsilon_0}}\sqrt{\frac{\mu}{\mu_0}} = -\left[\left(|\epsilon_r||\mu_r| - \frac{\epsilon''}{\epsilon_0}\frac{\mu''}{\mu_0}\right) + j\left(\frac{\epsilon''|\mu_r|}{\epsilon_0} + \frac{\mu''|\epsilon_r|}{\mu_0}\right)\right]^{1/2}$$
$$\approx -|\epsilon_r|^{1/2}|\mu_r|^{1/2}\left[1 + j\frac{1}{2}\left(\frac{\epsilon''}{\epsilon_r\epsilon_0} + \frac{\mu''}{\mu_r\mu_0}\right)\right] \tag{10}$$

Several experimental studies have been reported confirming this negative-index-of-refraction (NIR) property and applications derived from it, such as phase compensation and electrically small resonators, negative angles of refraction, subwavelength waveguides with lateral dimension below diffraction limits, enhanced focusing, backward-wave antennas, Čerenkov radiation, photon tunneling, and enhanced-electrically small antennas.
This studies rely heavily on the concept that a continuous-wave (CW) excitation of a DNG medium leads to a NIR and, hence, to negative or compensated phase terms.